P: ISSN No. 0976-8602 RNI No.UPENG/2012/42622 VOL.-V, ISSUE-I, January-2016
E: ISSN No. 2349-9443 Asian Resonance

On Common Fixed Points by Altering Distance between the Points

Khushal Devghare
Assistant Professor,
Deptt. of Mathematics,
Govt. J.H. P.G. College,
Betul, M.P.

Abstract

The aim of the present paper to obtain an answer to an open problem by using the relationship between the continuity of mapping in the setting of control functions which alter distance.

Keywords: Common Fixed Points, Control Functions, Compatible Maps, Complete Matric Space.

Introduction

The study of the existence and uniqueness of fixed points for self maps on a metric space by altering distances between the points with the use of control function.

Khan et al (1) established the existence and uniqueness of fixed point for single self maps. Sastry and Babu (2) proved a fixed point theorem by altering distance between the points for a pair of self maps.

Sastry et al (3) proved a unique common fixed point theorem for four mappings by using a control function in order to alter distance between the points.

The present paper aim to the open problem of Sastry et al (3) by obtaining a connecting between continuity of mapping in the setting of control function.

Definition 1.1

A control function $\varnothing$ is defined as $\varnothing$: $R^{+}\rightarrow R^{+}$ which is continuous at zero, monotonically increasing $\varnothing(2+)\leq 2\varnothing(+)$ and $\varnothing(+)=0$ iff $+=0$.

Definition 1.2

Two self mapping A and S of a metric space (X,d) are called weakly commuting if $d(ASx,SAx)\leq d(Ax,Sx)$ for each x in X such that

$$ASx=SAx \; whenever \; Ax=Sx$$

Definition1.3

Two self mapping A and S of a metric space (X,d) are called $\varnothing$ compatible.

If $\lim_{n\to\infty}\emptyset(ASx_{n},SAx_{n})=0$, whenever $\{x_{n}\}$ be a sequence such that $\lim_{n\to\infty}Ax_{n}=\lim_{n\to\infty}Sx_{n}=t$ for some t in X.

Definition 1.4

Two self mapping A and S of a metric space(X,d) are said to be reciprocally continuous in X. If $\lim_{n\to\infty}ASx_{n}=At$ and $\lim_{n\to\infty}SAx_{n}=St$ whenever $\{x_{n}\}$ is a sequence in X such that

$$\lim_{n\to\infty}Ax_{n}=t=\lim_{n\to\infty}Sx_{n} \text{ For some } t \text{ in X.}$$

If A,B,S and T are four self maps of(X,d) and $\emptyset$ is a control function on R^{+} then we have

$$M\emptyset(x,y)=max\{(\emptyset(d(Sx,Ty)),\emptyset(d(Ax,Sx)),\ \emptyset(d(By,Ty)),[\emptyset(d(Ax,Ty))+\emptyset(d(By,Sx))/2\}$$

Theorem

Let (A,S) and (B,T) be weakly commuting pairs of self mapping of complete metric space (X,d) and function satisfy

1. $AX\subset TX$, $BX\subset SX$ and
2. There exist h in [0,1] such that $\emptyset(d(Ax,Sx))\leq h\,M\emptyset(x,y)$ For all x,y in X and (A,S) or (B,T) is a $\emptyset$-compatible pair of reciprocally continuous maps then A,B,S, and T have a unique common fixed point.

Lemma

Let $f{:}R^{+}\to R^{+}$be increasing continuous at the origin and vanish only at zero then $\{t_{n}\}\subset R^{+}$ and f $(t_{n})\to 0$ implies that $t_{n}\to 0$.

Main Result

Theorem

Let (A,S) and (B,T) be weakly commuting pairs of self mappings of a complete metric space (X,d) and the function satisfy the condition

$AX\subset TX$, $BX\subset SX$ and

There exist h in [0,1] such that

$\emptyset\big(d(Ax,By)\big) \le hM\,(x,y)\ \forall\, x,y \in X$

Then the continuity of one of the mappings in $\emptyset$ -compatible pair (A,S) or (B,T) on X, implies their reciprocal continuity.

Proof

Suppose that A and S are $\emptyset$ -compatible and S is continuous then we show that A and S are reciprocally continuous.

Let $\{\ x_n\ \}$ be a sequence such that for some Z in X. We have $Ax_n \to z$, $Sx_n \to z.$

We show that

$\lim_{n\to\infty} ASx_n = Az$ and $\lim_{n\to\infty} SAx_n = Sz$

Now since S is continuous we get

$SAx_n \to Sz$ and $SSx_n \to Sz$ as $n \to \infty$

Also since (A,S) is compatible so we have

$Ax_n \to z$ and $Sx_n \to z$ implies

$\lim_{n\to\infty} \emptyset(d(\,ASx_n, SAx_n)) = 0$

And

$\emptyset\big(d(ASx_n,Sz)\big) \le \emptyset(d(SAx_n,ASx_n) + d(SAx_n,Sz) \to 0\ as\ n \to \infty$

Using following lemma

$d(ASx_n,Sz) \to 0)\ as\ n \to \infty$ and so

$\lim_{n\to\infty} ASx_n = Sz$

Also $AX \subset TX$ for each n. These exist w_n in X such that $ASx_n = Tw_n$ and $ASx_n = Tw_n \to Sz.$

Thus $SSx_n \to Sz$ and $SAx_n \to Sz$, $ASx_n \to S_n$ and $Tw_n \to Sz\ as\ n \to \infty$

Again we claim that $Bw_n \to S_n\ as\ n \to \infty$

Then there exists $\in > 0$ and $\{n_k\}$ such that $d\big(ASx_{n_k}, Bw_{n_k}\big) > \in$ And

$\emptyset\Big(d\big(SAx_{n_k}, ASx_{n_k}\big)\Big) < \in\ \forall\, n_k$

Therefore

$$\begin{aligned}\emptyset(\in) &\le \emptyset(d\big(ASx_{n_k}, Bw_{n_k}\big)) \\ &\le h\,M\emptyset(Sx_{n_k}, W_{n_k}) \\ &= h\max\{\emptyset(d\big(SSx_{n_k}, Tw_{n_k}\big)\}\end{aligned}$$

$$\{\emptyset\Big(d\big(ASx_{n_k}, SSx_{n_k}\big)\Big), \emptyset\Big(d\big(Bw_{n_k}, Twx_{n_k}\big)\Big),$$

$$[\emptyset(d\big(ASx_{n_k}, Tw_{n_k}\big) + \emptyset\frac{\big(d\big(Bw_{n_k}, SSx_{n_k}\big)\big]}{2}\}$$

$$= h\max\{\emptyset\Big(d\big(Bw_{n_k}, Tw_{n_k}\big)\Big), [\emptyset(\frac{d\big(Bw_{n_k}, SSx_{n_k}\big]}{2}\}$$

$$= h\,\emptyset\,(d\big(Bw_{n_k}, ASx_{n_k}\big))$$

$$< \emptyset(d\big(Bw_{n_k}, ASx_{n_k}\big))$$

which is contradiction.

Hence $\lim_{n\to\infty} Bw_n = Sz$ and

$\emptyset\big(d(Sz,Bw_n)\big) \le hM\emptyset(z,w_n)$

$= h\max\{\emptyset\big(d(Sz,Tw_n)\big), \emptyset\big(d(Az,Sz)\big), \emptyset\big(d(Bw_n,Tw_n)\big), , \frac{[\emptyset(d(Az,Tw_n)) + \emptyset(d(Bw_n,Sz))]}{2}\}$

Letting $n \to \infty$ we get

$\emptyset\big(d(Sz,Az)\big) \le h\max\{\emptyset(d(Sz,Az), \frac{[\emptyset(d(Sz,Az))]}{2}\}$

$= h\,\emptyset\big(d(Sz,Az)\big) < \emptyset(d(Sz,Az))$

which is contradiction.

Therefore we have $Az = Sz$

Thus $\lim_n SAx_n = Sz$ and $\lim_n ASx_n = Sz = Az$

This implies that A and S is reciprocally continuous in X.

References

1. M.S.Khan, M.Swaleh and S. Sessa ; fixed point theorem by altering distances between the points Bull. Austral. Mathe. Soc 30(1984), 1-9.
2. K.P.R.Sastry and G.V.R.Babu; some fixed point theorem by altering distances between the point Ind.J.Pure and Appl. Mathe 30(jun 1999), 641-647.
3. K.P.R.Sastry, S.V.R.Naidu generalization of common fixed point theorem for weakly commuting mapping by altering distance. Tamkang journal of mathematics 31(2000), 243-250.
4. R.P.Pant, K.Jha and S.Padaliya; common fixed point by altering distance between the points. Tampang journal of mathematics 34(2003), 239-243.